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ABSTRACT

In this study, children (ages 7-12) of average intelligence who had learning disabilities (LD) (n=29) and typical children (n=30) were individually tested in a task that investigated the development of proportional structures of thought. In addition, mathematical knowledge was assessed on the Woodcock-Johnson Tests of Achievement-Revised (WJTA-R). In this cross-sectional design, students in both groups coordinated increasingly more complex relationships among the elements of the problems as a function of grade. However, significantly fewer children with LD had yet constructed the second-order logical structures necessary to act on problems using multiplicative and pre-proportional reasoning. No children in either group demonstrated formal proportional reasoning, although a small minority evidenced qualitative proportional reasoning. On the applied problems test of the WJTA-R, the students with LD performed significantly below same-aged peers, although they achieved approximately at grade level on this task. Performance differences between the two groups on computation approached significance. For both groups, the explicitly taught procedures to solve computation and word problems as measured on the WJTA-R failed to represent accurately the degree of operational logic in children's biologically based structures of logical-mathematical activity. (Contains 35 references.) (CR)

Running heading: PROPORTIONAL STRUCTURES IN CHILDREN WITH LD

The Evolution of Proportional Structures in Children With and Without Learning Differences

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Abstract. Children in grades 2 and 4 through 7 of average intelligence who were (a) learning disabled (LD, n=29), and (b) not identified as learning disabled (NLD, n=30) were individually tested in a task that investigated the development of proportional structures of thought. Four of the children in the LD group were not classified but were receiving basic skill instruction due to their poor performance in mathematics both on standardized testing and in the classroom. Mathematical knowledge was additionally assessed on the Woodcock-Johnson Tests of Achievement-Revised (WJTA-R). In this cross-sectional design, students in both groups coordinated increasingly more complex relationships among the elements of the problem as a function of grade. However, significantly fewer children with LD had not yet constructed second-order logical structures necessary to act on problems using multiplicative and pre-proportional reasoning. No children in either group demonstrated formal proportional reasoning although a small minority evidenced qualitative proportional reasoning. On the applied problems test of the WJTA-R, the students with LD preformed significantly below same-aged peers, although they achieved approximately at grade level on this task. Performance differences between the two groups on computation approached significance. For both groups, the explicitly taught procedures to solve computation and word problems as measured on the WJTA-R failed to accurately represent the degree of operational logic in children's biologically based structures of logical-mathematical activity. Diagnostic and remedial implications are discussed.

The Evolution of Proportional Structures in Children With and Without Learning Differences

The evolution of proportional thinking in the field of learning differences (LD) remains largely unexplored. Further, in the limited research that has explored this area (Bley & Thornton, 1995; Carnine, 1991, 1993, 1997; Engelmann, Carnine, & Steely, 1991) the transmission of explicitly taught formulas in computations and word problems that are highly similar is emphasized, thus leaving virtually unexplored students' underlying mathematical structures of activity. Children's inability to compute ratio and proportion is attributed to disabilities in receptive and expressive language, abstract reasoning, and closure (Bley & Thornton, 1995).

Such a perspective is inconsistent with the theoretical perspective that the logic of proportional structures is not inherent in the numerical symbols and language used to "transmit" the meaning of the equation. Rather, the mathematical constructs of ratio and proportion are self-regulated biological structures of higher-order logical mathematical activity that owe their evolution to the recoordinations (i.e., simultaneous differentiation and integration) of organizing activity (Lamon, 1993; Piaget & Inhelder, 1975; Lesh, Post, & Behr, 1988). Because language is subordinated to this biological activity, language serves "to constrain and thus to guide" children's logical constructions (von Glasersfeld, 1990, p. 37) when used as a tool to expand children's internal organizing activity through conflict. However, it is not the empowering source of evolving higher-order structures.

According to Lesh et al. (1988), the most critical characteristic of proportional reasoning is the recognition of the "invariance of a simple mathematical system" (p. 101) where a "mathematical system" refers to biologically based organizing activity of logical-mathematical structures. The invariance of a mathematical system evolves over time by the alternating rhythm

of closure (differentiation) to gain stability in structures and openness (synthesis) to create increasingly more coordinated possibilities that are anticipated (i.e., reflectively abstracted) as structures reorganize onto higher levels (Piaget, 1987). In other words, it is through changes in how children coordinate unit structures in quantities (i.e., the "size chunks" assigned to the parts in relation to the whole) that the cognitive complexity involved in linking meaning, symbols, and operations, is accounted for (Behr, Harel, Post, & Lesh, 1994; Harel & Confrey, 1994; Lamon, 1993, 1994, 1996).

Accompanying the process of unitizing, and necessary to the construction of higher-order composite unit structures, is the process of partitioning (Pothier & Sawada, 1993; Lamon, 1996). Lamon (1996) argued that these two processes are interconnected and equally necessary for the construction of higher-order composite unit structures. Specifically, partitioning is an operation that generates quantity. It is an intuitive, experienced-based activity that serves to anchor the construction of rational number to a child's informal knowledge about fair sharing. Unitizing is a cognitive process that conceptualizes the amount of a given commodity or share before, during, and after the sharing process. According to Lamon, to understand how children come to understand rational numbers, we need to better understand the interaction of intuitive and cognitive processes that precede the construction of rational number quantity.

When logical-mathematical activity has evolved to additive structures of operational thought, children reflect on amount as a numerical composite whose parts are also composite units; that is, the sets that combine to form the larger set (Behr, Harel, Post, & Lesh, 1994; Lamon, 1994; Steffe, 1994). Because the composite units or sets are compared successively, the

inclusion relations between the sets can only be represented on one level of abstraction (e.g., one in two, two in three, etc.) (Clark & Kamii, 1996; Piaget, 1987).

For example, Clark and Kamii (1996) provided children in grades one through five with three fish in which the second and third fish were two and three times the size of the first fish respectively. The children are told, "This fish (pointing to B) eats 2 times what this fish (pointing to A) eats, and this big fish (pointing to C) eats 3 times what the little one (pointing to A) eats. . ." After one fish receives chips, children determine the amount of chips for the remaining two fish. Children with additive logical structures add a numerical sequence of +1 or +2 (A=7, B=8 and C=9) or +2 for B and +3 for C (A=7, B=9, C=10 or 12) because their structuring activity limits conceptual coordinations to a successive comparison of the composite units across one level of abstraction.

As children evaluate self-generated possibilities in tasks that provoke conflict, structures expand to such a degree that they are reorganized into second degree higher-order relationships and are referred to as multiplicative structures of thought (Clark & Kamii, 1996; Lamon, 1996; Piaget, 1987; Vergnaud, 1994). Advancement to this level of thinking involves the conceptual coordination of multiple composite units (Behr, Harel, Post, & Lesh, 1994; Lamon, 1994, 1996) such that simpler multiplicative structures require a three-tiered unit composition. In the fish task, if A=7 and B =14, the elements of 14 are coordinated as: (a) 14 (1 units), (b) 2 composite units each consisting of 7 1-units [2(7 units)]; and (c) 1 composite 2-unit consisting of 2 of the 3, 7 1-units [1(2(7 units))] or 2/3 of 21 (e.g., Lamon, 1994). Thus, in multiplicative structures of thought, the distributive property relates multiples of the same composite units where all of the relationships are considered simultaneously. Because two levels of abstraction are involved, a

second-order relationship is created (Clark & Kamii, 1996; Piaget, 1987; Lesh, Post, & Behr, 1988).

What differs between the simpler multiplicative structures and the more complex structures of proportional reasoning is that proportional reasoning involves an abstraction between two second-order relationships simultaneously rather than a relationship between two concrete objects or two directly perceivable quantities (Piaget & Inhelder, 1975; Lesh, Post, & Behr, 1988). Thus, the student cognitively coordinates multiples of different composite units (Lamon, 1994) and reflects upon the equation A/B = C/D as a dynamic transformation where the structural similarity on both sides of the equation is attended to. This coordinated abstraction between the two complex systems of relationships enables the child to change any element of an equation to compensate for a change in another element. Lamon (1994) referred to this process as using the same scalar operator (i.e., ratio) in both measure spaces of the equation such that ratio is invariant across situations (Confrey & Smith, 1995).

If there is lack of evidence that the child's mental structuring activity coordinates the structural similarity on the two sides of the equation (i.e., the equivalence of appropriate scalar ratios and the invariance of the function ratio between two measure spaces), then there can be no claim for proportional reasoning (Lesh et al., 1988; Lamon, 1993; Piaget & Inhelder, 1975). For example, if the child believes the pieces of food can be arranged into different groupings for each fish, then the abstraction is non-reversible or pre-proportional (i.e., the child lacks the ability to deduce that a change in one of the four variables in a proportion changes the remaining variables) (Piaget & Inhelder, 1975). This type of pre-proportional thinking is most evident when tasks are related to an algorithmic solution (Lesh et al., 1988).

Confrey (1994) and Confrey and Smith (1995) placed a greater emphasis on the role of splitting in the development of ratio than previous authors (e.g., Lamon, 1994; Lesh, Post, & Behr, 1988) while further asserting that ratio co-evolves with multiplication and division rather than being an outcome of abstract mathematical operations. Specifically, the early acts of splitting such as sharing, dealing, magnifying, and creating multiple sets of equal groups, lead to the development of a splitting structure (i.e., multiplicative structure). For example, in a splitting structure of two, a movement in one direction is doubled and movement in another direction is halved. This repeated action is preserved by reinitializing (i.e., the process of treating the product of a splitting process as a basis for the reapplication of that process) where the origin is always one. The unit is therefore the invariant relationship between a predecessor and a successor in a sequence that is formed by repeated action.

The primary purpose of this study is to examine the evolution of proportional structures of thought in children with LD. Of particular interest is whether these children will display a "disordered" pattern of development that would indicate the presence of a specific disability. In a pendulum problem (Piaget & Inhelder, 1958) requiring the use of ratio for correct problem solution, children with LD "actively" developed strategic thought problems, albeit differently (Swanson, 1993). These differences, according to Swanson, are related "to their metacognitive ability to use other strategy subroutines in a flexible manner" (p. 885). The authors discussed in this manuscript argued that strategies generated (i.e., possibilities) are indicative of the quality of children's organizing activity (i.e., logical-mathematical structures). If LD children use strategies that typify younger NLD children, then the use of less sophisticated strategies resides in thought structures that are not yet coordinated onto higher-order levels.

A second purpose is to compare the performance of all children on the fish problem to a traditional mathematics task. If children display differences in operational reasoning on the fish task from procedures that are explicitly taught to solve computation and word problems irrespective of their coordinating activity, then learners become rigid in their thinking with limited ability to generalize this learning to real-life situations (Smedslund, 1961; Sinclair & Sinclair, 1986). Such dynamics would explain the lack of flexibility in the use of strategy subroutines in LD children (Swanson, 1993). (See Mastropieri, Scruggs, & Butcher, 1997 for the poor transfer ability in LD children when explicitly taught strategies removed from organizing activity are taught in a pendulum task.)

The third and final purpose is to examine the relationship between what Kieren (1994) defined as the splitting-analytical and distributive-algebraic aspects of multiplicative structures in the evolution of proportional structures for all students.

METHOD

Subjects

A total of 62 children participated in the study during a one-month period in December. Students were grouped into one of three categories: (a) students classified as learning disabled (LD) due to a discrepancy between language performance (reading and writing) and IQ in the average range; and (b) students in basic skills instruction (BSI) for mathematics because their total mathematics performance was below the 45th percentile on the Comprehensive Test of Basic Skills-4th edition (CTBS). The mean percentile of the four students combined in BSI for total mathematics performance is 33.75. The four students who qualified for BSI were all in grade six and entered the sample through random selection. They were receiving additional

instruction in mathematics either at home or in the school. These four students were placed in the group of students with LD.[1] Eighty-six percent of the children with LD were receiving support or replacement services for mathematics as detailed in the text that follows.

The remaining student type consisted of students with no history of learning problems (NLD) and comprised the group with NLD. Multiple indexes (cognitive scale indexes (CSI) on the CTBS almost two standard deviations above the average range (124-130) and Mahalanobis' D approximating 4) led to the decision to drop three of the children with NLD (one each in grades 4, 6, & 7) from the sample. The majority of the students who remained in the sample were in grades 5, 6, and 7. For these three grades respectively there were 10, 10, and 5 children with NLD and 8, 11, and 6 children with LD. Thus, there were an equal number of children in these three grades combined (25), although there were more children with LD than NLD in the sixth and seventh grades. There were a very small number of children in grades 2 and 4 for both groups. Specifically, there were two children with LD and NLD in grade 4. In grade 2 there were two children with LD and three children with NLD.

Table 1 presents descriptive information on age and standardized testing for children with and without LD. The mean ages for children with NLD in grades 2, and 4 through 7 respectively were 7.8, 9.9, 11.2, 11.8, and 12.6 while the mean ages for children with LD in the same grades were 7.7, 9.5, 11.1, 11.8, and 12.5. Age was not a significant difference between student groups. This table also shows that with the exception of the seventh grade, the majority of students in both student groups were males.

(Insert Table 1 about here)

All children in both student groups had cognitive aptitude scores within the low average to high average range (85-115). The cognitive scores for the children with LD were from individual testing done by a certified school psychologist using the WISC-III (test readministered every three years). For the children with NLD, the CSI on the CTBS were used. There were no CSI scores for the children with NLD at the second grade because the school system did not test students until the end of that grade. The mean Full Scale IQ scores for students with LD in grades 2, and 4 through 7 respectively were 96, 102, 94, 104, and 103. For children with NLD the IQ scores were 108, 104, 110, and 108 for grades 4 through 7.

The reading/word decoding test on the Wide Range Achievement Test (WRAT) (third revision) was administered by the author using standardized directions as a general screening of children's grade level in word recognition. Table 1 shows that the mean standard score performance for children with NLD in grades 2, and 4 through 7 respectively was at or slightly above expected grade level performance: 107, 109, 101, 109 and 110. In contrast, the mean standard score performance of children classified as LD was generally below expected grade level performance, but within one standard deviation of the mean: 93, 88, 83, 92, 99 for children in grades 2, and 4 through 7 respectively. All but two of the children with LD attended a resource room or self-contained class for reading and language arts at the time of testing as shown in Table 2. Specifically, there were two seventh-grade children with LD who attended resource room for support only due to significant progress in language related areas.

The number (and percent) of children at each grade level receiving remedial intervention in mathematics (support, basic skills, or replacement) was: one (50%); one (50%); seven (93%); eleven (100%); and five (83%). Thus, most of the children who participated additionally showed significant evidence of difficulty in mathematics by their classroom performance and/or in standardized testing. (See Table 6 for performance scores in mathematics.)

(Insert Table 2 about here)

Task Materials and Scoring

Fish Task

The fish task used in this task (Clark & Kamii, 1996) was a modification of a task devised by Sinclair (Piaget, Grize, Szeminiska, & Bang, 1977). In the study by Clark & Kamii (1996) children in grades one through five were tested. The children in this study went up to the seventh grade because previous research indicated that children with LD were delayed with regard to the evolution of multiplicative structures (Grobecker, 1997). The materials consisted of 3 "fish," 5, 10, and 15 cm long, made of white oat tag paper with black drawings on them representing the features of the fish (see Figure 1) and about 100 cheerios in a plastic tupperware bowl. The "fish" resembled eels that vary in length but not in any other dimension.

(Insert Figure 1 about here)

The procedure for administrating the task was as follows (step 4 was added by this author but otherwise the 5 steps were the same as Clark and Kamii, 1996):

1. The fish were placed in front of the student as shown in Figure 1.

2. The child was told, "This fish (pointing to B) eats 2 times what this fish (pointing to A) eats, and this big fish (pointing to C) eats 3 times what the little one (pointing to A) eats. This fish (B) eats 2 times what this fish (A) eats because it is 2 times as big as this one (A)." The interviewer demonstrated by showing that A could be placed on B two times. Then the interviewer continued, "The big fish (C) eats 3 times what the little fish (A) eats because it (C) is 3 times as big as this one (A)." The interviewer again demonstrated by placing A on C three times.

3. One cheerio was fed to fish A and the first question was then posed to the child, "If this fish (A) gets 1 cheerio how many cheerios would you feed the other two fish? Remember that this fish (B) eats 2 times what the little fish (A) eats, and the big fish (C) eats 3 times what the little fish eats." Similar questions were then asked with the following variations: (2) when B received 4 cheerios (2, **4**, 6); (3) when C received 9 cheerios (3, 6, **9**); (4) when A received 4 cheerios (**4**, 8, 12); and (5) when A received 7 cheerios (**7**, 14, 21). One LD and NLD student in grade six were asked to write a number fact to problems 3 and 4 respectively in an attempt to better understand why they were having difficulty grouping the cheerios.

4. If the child failed to independently group the cheerios they were asked, "Is there any particular way you could put the cheerios to show that this fish (B) eats two times what this fish (A) eats and this fish (C) eats 3 times what this fish (A) eats?" Because this question was initially not planned for but added as a result of children's actions, there were variations of this question

asked. If the child looked puzzled, other suggestions were given such as putting the cheerios into groups.

5. If a child answered problem three incorrectly (3, 6, 9), a counter suggestion was offered: "Another girl/boy told me that if this big fish (C) gets 9 cheerios, the little fish (A) should get 3 cheerios because 9 (pointing to the 9 cheerios rearranged into three groups of 3) is 3 times what this is (pointing to the 3 cheerios given to A and arranged in one group). And this fish (B) should get 6 cheerios because 6 (pointing to 6 cheerios rearranged into two groups of 3) is 2 times what this is (pointing to the 3 cheerios given to A). What do you think of his or her idea?" After the child gave an opinion, the interviewer asked for an explanation: "Why do you think the other person's way is better?" Other questions often followed based on the child's responses to probe the child's reasoning. In the counter suggestion care was taken to say, "Nine is 3 times what this (A's cheerios) is," rather than "Nine is 3 times what three is," to avoid suggesting the multiplication tables. The children were also probed about the grouping of the cheerios.

Because the purpose of the study by Clark & Kamii (1966) was to distinguish additive from multiplicative structures, these authors did not discriminate higher levels of multiplicative thought in terms of how the children were grouping the "food" for each fish. In this study, children's justifications of the grouping relationships made were probed in an effort to better define the progressive evolution of multiplicative thought structures.

Another component was additionally added to the task. Specifically, after the last problem above was given, the child was told, "Now we are going to do something different. I'm going to feed a fish some cheerios and you have to write a number problem to figure out what the other two fish eat. You can use this paper and pencil to help you." The child also had to give

justifications for the problem written. Clinical interviews followed to better understand children's reasoning. If they couldn't write a number problem, the children were given a quick glance of the first page of the computation test on the WJTA-R to provide a familiar example. The three problems given to the children were: (1) when C received 15 cheerios (5, 10, **15**); (2) when B received 12 cheerios (6, **12**, 18); and (3) when A received 3 cheerios (**3**, 6, 9).

The children were then administered the computations and applied problems tests on the WJTA-R and asked to think out loud as they solved the problems on both tests. A very small number of children did not verbalize their thoughts when they solved the number problems but all thought out loud when doing the applied problems. The experimenter wrote what the children said and did and audio-taped them as well.

After all children were tested, the protocols were scored. The levels that were scored were based on the descriptions of the levels by Clark & Kamii (1996). However, because additional attention was given to the manner in which the cheerios were grouped for each fish as well as the fact the children exceeded the fifth grade level, the levels for multiplicative thinking were modified. Specifically, a level for doubling was added and correct multiplicative solutions were subdivided into incorrect and correct groupings for each fish. Seven levels in the evolution of additive to multiplicative logical-mathematical structurations are described in the text that follows. The scoring of the levels in the problems with cheerios was checked in seventeen randomly selected protocols by a mathematics professor who was familiar with the study. She was unaware of student group or grade. In 82% of the cases we came to agreement on the original scored level. The remaining protocols were settled by discussion.

RESULTS

Fish Problems with Cheerios

There are seven levels coded. The first four levels are indicative of additive structures of thought. These levels consist of: (I) no serial correspondence; (II) numerical sequence of +1 or +2; (III) additive thinking involving +2 for B and +3 for C; and (IV) transitional additive to multiplicative. The remaining three levels describe multiplicative thought structures: (V) doubling; (VI) all correct but lack of logical consistency in grouping the cheerios; and (VII) all correct with logical groupings. In a description of the levels to follow using student protocols, the bold numbers indicate the number of cheerios the interviewer gave to a fish and the regular numerals show the number of cheerios given by the children. The data were analyzed both qualitatively and quantitatively.

Qualitative Analysis.

Table 3 shows the number (and percent) of students at each grade level for both student groups that achieved at each of the seven levels. As grade increased, the children with NLD were more consistent in achieving at the higher levels of multiplicative thought. A description of each level follows using protocols of children to describe the levels.

(Insert Table 3 about here)

Level 1: No serial correspondence or serial correspondence with qualitative quantification. The children who seriate in this task think only qualitatively in terms of "more" or "less" and accept almost any number as long as $A<B<C$.

(Grade 5, LD) For 4 to B, he gives C 7, changes the amount to 8, then gives A 2. (2, 4, 7→8) *"What do you have there?"* "Eight." *"You put 8 instead of 7?"* "Yeah." *"Why did you feed this LF 2?"* "Because it's like almost its size." *Why?"* "'Cause it's like almost its size and its stomach isn't really that big." *"It's almost the same size?"* "Yeah." *"Why did you feed this BF here 8?"* "That one got more. I thought it could eat more. Maybe it could eat more. Yeah, it has a lot of room." *"How many more did you give it?"* "It ate one more. I gave it like about 3 more." *"Three more?"* "I thought like this one (B) got 4 so I thought maybe it could get a little higher." *"So you just made it higher?"* "Yeah." *"And you picked, how many more did you pick to decide to make it 8?"* "Three more." He places the cheerios randomly on and under the fish. *"Why did you put them that way?"* "Because it shows the size. It's something small."

When given the counter (i.e., the correct solution), two students think either answer is acceptable but state justifications unrelated to the mathematical logic of the problem. One student (Grade 5) prefers the counter using an explanation that is consistent with the logic of the problem. However, on the two problems that follow, he regresses to his previous nonlogical patterns of thinking. None of the students place the cheerios in meaningful groupings.

<u>Level II: Additive thinking with a numerical sequence of +1 or +2.</u> The children at this level relate A to B and B to C, and give B one or two more than to A, and give to C one or two more than to B. A small number of students use reasoning suggestive of attention to the splitting of groups, but then add or subtract rather than dividing the groups. The second protocol is of a student with LD who adds prior to the counter but then thinks that it is correct to add or multiply after the counter. As demonstrated by his lack of ability to group the cheerios in any meaningful manner and reflect on his actions, his understanding of multiplication is purely rote.

(Grade 2, NLD) For 9 to C she feeds B 8 and A 7 (7, 8, 9). She places the cheerios in two uneven lines under B and one line under A. "*OK, how come 8 for the MF?*" "Because if this (C) was 9 that would if that one (B) would be 8. This would have to be 9 x it." *"Ok, that makes it 9 x it?"* "This (B) would be 8 and I guess you would take away 1 because this fish (C) eats 3 and this (B) eats 2, and this eats 1." *"Ok, and how come you decided 7 for that fish (A)?"* "Because if this (B) was 8 and this (A) would be 7 because this this is 7 and this would be 7 x as much as 8." *"Would you leave the cheerios like this under the*

> *big fish?"* "They're alright." After the counter she responds, "I wouldn't do that." *"Why not?"* "Because 9 this would be 9, and then this (B) would be 8 and this (A) would be 7." *"So you think she's wrong then?"* "Yes."

> (Grade 5, LD). This student consistently adds 1 (subtracts 1) to each fish on the first two problems (3, **4**, 5; 7, 8, **9**). After the counter he states, "It's good." *"Think it's good?"* "Yeah." *"How come?"* "Because that's the thing like this one (B) is 2 x this (A) and this (C) is 3 x that (A). Ok, times tables." *"Do you think that one solution is better than the other or are both ok?"* "Both ok." *"How come?"* "I don't know. I just think they're both ok." In the next problem immediately following the counter, he adds +1 (**4**,5,6) "Because . . . it's in order." *Why 6?"* "Because this one's bigger than all of them so he should get the most." On the last problem, he comes to the correct solution (7, 14, 21), but his justification does not exceed rote memorization of the x tables. "This time I'm going to do it the other way. Fourteen cause it's 2 x." *"Two x what?"* "The LF. And this one (C), this one gets 21 cause it's 3 x the LF, 21." He places the cheerios randomly in one large group. *"Is there any particular way to put the cheerios to show that this (B) is 2 x what the LF eats and this (C) is 3 x what the LF eats?* "Yeah, I did. I put in 21. This one has 7 and 7 x 3 = 21 and 7 x 2 =14." There is no grouping of the cheerios.

Only three of these students place the cheerios in groupings in some systematic manner. Two of the students group the cheerios in even number problems in twos and leave the problems with the odd numbers without groups. One student puts the cheerios in different groupings under each fish only after the counter, returning to random grouping in the last problem.

In response to the counter, the majority of students think either solution can be used and two prefer their own. Only one student (grade 6, LD) offers a rationale to the counter, although his reasoning is limited:

> "The other one cause I think it's correct." *"Why?"* "Cause there's this one (C) eats 3 times more 3 more cheerios than the larger one cause its small." *"The middle one?"* "The MF does the same thing but it eats 3 more than the smaller one." *"Why?"* "Cause it's the smallest one." This student continues to add +2 in the problems that follow.

<u>Level III: Additive thinking involving +2 for B and +3 for C.</u> The distinguishing feature of Level III is that the children take into consideration the number of "times" the interviewer stipulated; that is, 2 times A for B, and 3 times A for C. Most of the students are considered to be

transitioning into this level because they do not use this strategy for all problems and sometimes demonstrate a degree of confusion when this strategy is used. The majority of students use a combination of additive and multiplicative terms, but similar to the students in the previous level, add when stating that a fish ate 2 or 3 times more.

> (Grade 4, NLD). For 4 to A, this student ends with the solution of 6 and 9 to B and C respectively (4, 6→7→6, 10→9). "I would give this MF 6. No, I'd give that fish (B) 7 then I'd give this (C) 10." *"Why the MF 7?"* "I'd give the MF 7 'cause we gave the LF 4 and the 4+3 is 7. The MF would have 3 more. *"The MF would have 3 more?. . . "* "Was this one (B) suppose to get 2 more?" *"Two x as much."* "Oh, them I'd give this (B) 6.Then give this only 6 and then I would add 3 more than 6 to the BF." *"Why 3 more to the BF?"* "'Cause its 3 x bigger. . .If it goes like 2 x this guy (A) this one (B) gets 2 more. It's like 3 x its size so it got 3 more." *"And you want the cheerios left like that to show this (B) is 2 x what the LF eats and this (C) is 3 x what the LF eats?"* "Yes." The cheerios are placed randomly under the fish.

In partitioning the cheerios, approximately two thirds of the students have no apparent mathematical logic to their placement although one student attempts to put them into twos or threes in the first problem after the counter. The remaining students make at least one attempt to put the cheerios in groupings that show an additive relationship and progress to making multiplicative groupings after the counter. However, these groupings were not indicative of the composite unit structures of the problems.

The majority of children think that both solutions are equally good when the counter-example is provided. Two out of the three students who believe the counter-example is better fail to demonstrate a improved understanding in their justification or in the problems that follow.

> (Grade 5, LD) "Oh yeah, I think it's even better." *". . . Why?"* "'Cause it makes more sense." *"Why. . .?"* "Because that before this one (A) this one only ate 2 add 1 more is 3 and then this one (B) only ate 5 and 4 add 1 more is 6. And this one (C) only ate 8 before and 1 more is 9. And so if they all add up like that then this one (B) would go 2 less and I mean 3 less and 2 less like 3, 2 'cause this (C) is 3 x less than this (B) and 3 of that (A) and this is 2 of that (A).

Level IV: Transitional Additive to Multiplicative. In at least one problem with the cheerios, children use an additive solution involving +2 for B and +3 for C where they take into account the number of "times" the interviewer stipulated. However, rather than multiplying, they add. In at least one other problem, students use the logic of multiplicative thinking. With some children, inconsistency between language and logical structures persist.

> (Grade 5, NLD). For 9 to C, she feeds A 5 and B 7 {5, 7, 9}. This one {B} would get 7 and this one {A} would get 5. *"Ok, go ahead and feed them."* She places them in groups of two with one left over. *Why the MF 7 cheerios?"* "Well, because if the big fish has 9 then this one {B} would have 2 less because it's not another half . Doesn't have the other part of the LF." *"Why the LF 5 cheerios?"* "Because it's only half of the little fish so it would get 2 less." *"So it shows 2 x what the LF eats and this {C} is 3 x what the LF eats?"* "Yes." This student prefers the counter-example (provided below) and the problems following are correct, albeit incorrect groupings of cheerios.

Additionally included in this group is one sixth grade student with LD who demonstrates multiplicative thinking throughout all of the problems with the cheerios, but then regresses to the strategy of +2 or +3 for the problems without the cheerios. (Protocol later provided.)

Approximately half of the students (mostly LD) at this level randomly group the cheerios. One student (grade 7, LD) puts the cheerios into a smilely face after the counter-example because, "It looks neat. Just 'cause it looks neat." Four fifth grade students with NLD partition the cheerios, but these groupings are not always consistent with the composite unit structures of the problem. Although over half of the students agreed with the counter-example, the students with NLD tended to "assimilate" the logic as demonstrated in the two problems that followed.

> (Grade 5, LD). "I think it's a really good idea because that'd be like 9 and then you'd break down 2 but if you had to break down 1. I think it's a good idea too." *"Do you think hers is better?"* "Probably easier. 'Cause if I gave it to my cousin she'd probably break it down like my cousin probably."

(Grade 5, NLD-previous student described above). "I think they way she did it worked out better." *"Why?"* "Because mine is like they're odd and it's not as much." *"You think they should have as much?"* "Yeah." *"Why?"* "Because the way the problem would work out. The 9 should go down into like the next number and you divide."

Level V: Doubling. The children at this level consistently demonstrate understanding that number amounts consist of second-order composite unit groupings, but they direct much of their attention to the action of splitting by halving or doubling these units rather than distributing an equivalent unit across the fish n times. Further, even with the correct numerical solution, the great majority of children fail to distribute equivalent unit groupings of cheerios across the three fish n times.

(Grade 6, LD). For 4 to A, he feeds B 8 and C 16 (4, 8, 16). "This one (B) 8." *"Why?"* "Because you doubled the 4. And you give this one (C) 16 'cause you double the 8." He put all the cheerios in one group under each fish. *"Any way the cheerios should go under the fish?"* He rearranged them into a row of 4 under A, two rows of 4 under B and two rows of 8 under C. *"Why in rows like that?"* "No particular reason." *"So this fish (B) eats 2 x what the LF eats and this fish (A) eats 3 x what the LF eats?"* "Yes."

One child (grade 4, NLD) has all of the numerical solutions correct in the problems with the cheerios, but then halves or doubles the amounts on the first two problems without the cheerios and does not correct them as she reflects on her work. Additionally, she does not correctly group the cheerios. For this reason, she was put at this level.

The great majority of the students like the counter solution better. For those students who get the correct answer but the wrong groupings, all prefer the correct groupings. However, their justifications show attention mainly to the ease of counting and/or to the fact that the groups could be subtracted or divided evenly. Thus, there is no evidence that they assimilated higher-order composite unit structures from the counter.

> (Grade 5, NLD) "Oh, so it goes like by odds." *"What do you think about his answer..."* "Yes, I agree with it." *Better than yours?"* "Yes." *"Why?"* "Because it was more exact than what I said." *"Why. . .?"* "Because the SF probably would eat 3 instead of 1 ½ And the MF would probably eat 2 x as much as the smallest and the big fish would eat 3 x as much as the smallest fish."

Level VI: Correct problem solutions but lack of logical consistency in grouping the cheerios. Three students (grade 6, NLD) were transitioning into this level due to their tendency to want to double the amounts between fish. However, as they solve the problems, they work through this doubling tendency to have all problems with and without the cheerios correct. There is one exception to this rule. Specifically, when a student with NLD in grade seven was given 4 to B 2, **4**, 6), she multiplies 4x3 to get 12 for fish C. However, because she does not double the amount and got all of the remaining solutions correct, she is placed at this level. To better understand why these students are having so much difficulty grouping the cheerios, I probed the thinking of a sixth grade child with NLD further. This student is transitioning into this level because he initially attempts to halve an amount on one fish problem without the cheerios but then changes his thinking as he reflects on his work.

> For 4 to A. . .*"What about the third fish?"* "He has 12" (**4**, 8, 12). He then rearranges the cheerios into all different groups. He does not maintain the groups of four. *"Give me a number problem to show me why the fish has what you gave him."* He wrote 8+4. *"Why?"* "Cause the 8 from this (B) and then plus 4 from this one (A) equals 12." *"So how should the cheerios by arranged?"* First he made three groups of 4. He then changed it to 1 group of 8 and 1 group of 4. *"Does that show this number problem?"* "Well, if you add these two (A & B), you get the number of what this one is (C)." *"So you think they should stay like that?"* He puts them back into three groups of 4. I point to 8+4 and ask if this is his number problem. "No." *"Can you show me a number problem to show how you grouped them?"* He wrote 4x3. *"Is this number problem better?"* "Yes, 'cause more detail. . . Like it tells you more about it. Shows you that you can break it into 4 groups of 3 to get 12 cheerios."

The protocol to follow is of another student transitioning into this level. This student's thinking provides significant insights into how the processes of the distributive-algebraic and the splitting-analytic process are interdependent and supportive of each other. Specifically, for problem 4 (4, 8, 12), she clearly thinks about four higher-order composite units as noted by her reference to an "extra one" as a unit of four. However, she reflects on and modifies her reasoning while acting on the splitting structure of 4.

> (Grade 6, NLD) For 4 to A, she initially lays out 8 to B and 16 to C and then changes C to 12 as she reflects on the problem (4, 8, 16→12). At first, the cheerios are randomly placed. *"So that's (C) 16 there?"* "Yes." *"How are you going to fix those?"* She begins to lay the cheerios out in groups of 4. "Oh, wait, I have an extra one." She takes away the extra 4 from C. *"Why do you think you had 16 first?"* "I quickly thought that I thought 4 x 4 =16." *"You thought 4 x 4?. . . Why . . .?"* "It's one of them that I know very well"... *"Why do we have them the way you put them out?"* "When you say that this one (A) would get 4 then that one (B) double this one (A) and this one (C) gets triple this one (A)."

Most of the students who are given the counter example due to incorrect grouping of the cheerios, like the groupings better and tend to correctly group the cheerios after the counter. However, the grouping they prefer is generally due to the fact that the they make the cheerios easier to count and not due to a more coordinated abstraction of the problem components.

> (Grade 6, LD). *"What do you think about his idea?"* "It's good." *"Why?"* "'Cause he explained it nicely and he's right." *"Do you think your fish should have the cheerios like that or do you want to keep them like that?"* The cheerios were randomly distributed and he rearranged them into groups of 3. *"Do you think that's better?"* "Yes." *"Why?"* Because now they're like all in rows and you can count easier." *"Ok, how would you count?"* "You count in rows of 3."

<u>Level VII: Multiplication all solutions and groupings correct</u>. Students' logic is concise, expressing only pertinent points while accurately representing multiplicative terms. However, there is a tendency for approximately half of the students to believe that other groupings do not distort the problem. Thus, they appear to have an intuitive sense of invariance of groupings

across the fish, but the structures for proportional reasoning are not yet fully evolved and stabilized.

> (Grade 6, NLD) For 7 to A, she feeds B and C 14 and 21 (7, 14, 21)... *"I see you have them in groups of 7. Would it make sense to group them any other way and still have this (B&C) being 2 x and 3 x as big as this one(A)?* "I guess you could group them in twos if you want to count in twos. But it would take a longer time than if they were in sevens." *"If you did that, would that still be 2 x what this LF eats?"* "Yeah, they're just grouped differently." *Why don't you try that?"* These two (A&C) wouldn't work with this 'cause they're odd numbers and this one wouldn't work because 14 is an even number." *"What would your times problem be though, to show that this (B) is 2x what this (A) eats if you group them like that?"* "I guess it wouldn't cause it's hard to find a way to make it so it looks like its 2 x bigger this one (B) gets 2 (groups of 7). 'Cause these two (A & C) are odd. So they can't go in groups. Like they're hard to chop."

Fish Problems Without the Cheerios.

The quality of number problems written by the children was investigated within each of the above seven levels in the task with cheerios. There are performance patterns unique to the first four levels describing additive structures of thought and the last three levels describing multiplicative thought structures. For this reason, the descriptions of children's performance are discussed according to additive or multiplicative thought structures.

Additive structures of thought. The great majority of children at this level require varying degrees of questioning to help them write number problems to represent their thinking although this help generally decreased with the two problems that followed. It was necessary to give seven students a brief glance of the computation part of the WJTA-R to help identify what a "number problem" is. When these students add or subtract, they tend to follow the same pattern in how they feed the fish with the cheerios. For example, if they add/subtract one cheerio from each fish at level II, then they add/subtract that same number in the written problems. However, within each of these levels, there are also students who write a number problem to get a desired answer

that doesn't reflect the logic they are representing in problems with the cheerios. Similar to the problems with cheerios, if multiplicative terms are used to describe the problem written, the terms represent additive constructions in the child's mind.

> (Grade 5, LD). Fifteen cheerios are fed to C. After being shown the WJTA-R and redirected to write a problem instead of using the cheerios he responds, "I would probably give this one (B) about 10." *"How'd you figure that?"* "Cause uh no, not 10. I'd probably give it 13 cheerios. And should I do something?" *"How'd you get 13?. . . What kind of number problem would you have?"* "Like 15-13 or something?" *"Whatever you're thinking."* "Would be 2." (Wrote 15-13). "So it would be 2 and so I would say this one (A) would get 11." *"How'd you get 11? Write a number problem."* "Like ok, so 13+11. I'd get 24." *"Ok, which one would get 24?"* "Well, this one (B) probably." *"So this one (A) would get 2 and this one (B) 24?"* "No, I thought you meant how much this one (B) would get and how much this one (A) would get." *"That's what I want to know."* "This one (B) would get 13 and this one (A) would get 11." (11, 13, **15**) *"So show me with a number problem how you got the answer of 13."* "Well, alright. 15-3." "And then I'd put 15-2 and put the 1 on top would be 13. So that's the MF." *"How'd you get 11?"* "Well, 11. That would be 13-2. And I'd get 11." This student achieved a standard scored of 97 and 112 on calculation and applied problems respectively.

> (Grade 5, LD) This student demonstrated similar difficulties in the first problem but quickly computed 12+3 and 12-2 for the second problem (10, **12**, 15). *"Wow, you did that really fast."* "I get it now. Before I didn't know how to do those things."

> (Grade 2, NLD). For 12 to B he wrote 12-2=10. *"Why did you take away 2?"* "Because that one (B) is bigger than that one (A) and eats 2 x as much as that one." *"Ok, what about your number problem for the other one?"* Wrote 15-3=9 and erased it. He then wrote 15-3=12. (10, **12**, 15) *"Why 15-3?"* "'Cause that one (C) eats 3 x more than that one (B).

Some of the students at level IV begin to direct their attention to the splitting of groups when computing and explaining their answers which is accompanied by several errors in their thinking as shown in the first protocol to follow. The second protocol is of the previously discussed sixth grade student with LD who gets all problems correct with the cheerios but then consistently adds/subtracts 2 or 3 in the problems without cheerios.

> (Grade 5, NLD) For 12 to B, "So it could be in 3 groups of 4. I would feed. . ." *"Show me a number problem."* "Oh, if 15-4 wait. How many is there, 15?" *"No, this fish here eats*

> *12."* "Twelve. This fish (A) would eat 8 cause if you put them into 3 groups of 4 and you take away like one group it would be 8 so that's how much this one would be and then if you have 12+4 this (C) would eat 16. (8, **12**, 16) "'Cause if you add one more group it would be 16 cause this one eats 12 add 1 more group of 4 it would be 16."

> (Grade 6, LD) For 12 to B this student writes 15-13=3, erased it, then wrote 15-3=12 and 12-3=9 (9, **12**, 15). *"Ok, what do you have there? Why that number problem?"* "This has 15 right?" *"Yeah."* "Well, I subtracted 3 from this (C) and get 12 and I subtract 3 from 12 and get 9." *"Ok, why did you subtract 3?"* "Because this guy (B) eats double what this guy eats (A). *"I'm still unsure why you subtracted 3. Is there any other explanation?"* "Not really."

Multiplicative structures of thought. Children's number problems at this level indicate increasing sophistication in how they reflect on their learning activity as they advance from doubling to the more advanced levels as indicated in the protocols to follow.

> (Grade 6, NLD). For 15 to C, he writes 15/2=7 ½ . *"Ok, why did you do that?"* "'Cause this (B) eats 2 x the SF and this one (C) eats 3 x the SF so I divided these two." . . .*"What about the SF?. . ."* "I would also divide here." He wrote 15/1=15. "Hum. . ." Then he wrote 15/3=5. "So I would give this SF 5 this one (B) 7 ½ and this one (15). " *(*5, 71/2 **15**).*"Why did you divide that like that?"* "Because this (C) eats 3 x as much as this one (A) and this one eats 15 so I divided 3 by 15."

> (Grade 6, NLD). For 15 to C. She writes 15/3=5 and 5x2=10. (5, 10, **15**). *"Why divide by 3?"* "Because that would give you the answer to the LF because it's 3 x bigger." *"OK, and why 5x2?"* "Because the LF gets 5 and the MF is 2 x bigger."

Quantitative Analysis for Problems with and without Cheerios.

The children in this sample did not begin to demonstrate multiplicative thought structures until they were in grade five. Due to this observation, as well as the fact that there were a small number of students at the second and fourth grade levels, differences in type of thought structures used (additive versus multiplicative) were examined only for students in the fifth through seventh grades. An ANCOVA was conducted on the dependent variable of level achieved on the fish task

using IQ as a covariate and group as a factor. Significant differences were found for group [F (1, 47)= 5.36, p<.02] with more NLD students achieving at the level of multiplicative structures.[2]

The number (and percent) of students achieving success on each of the five number problems with cheerios for all students is presented in Table 4. Table 5 shows the number (and percent) of students achieving success without cheerios on the three number problems. I failed to give one sixth grade student with NLD problem 5 with the cheerios and one fifth grade student in the same student group problem 3 without the cheerios.

(Insert Tables 4 & 5 about here)

For the five problems with the cheerios, significant group differences were found for the following three problems using Pearson's crosstab analysis: three (3, 6 ,9), $x^2(1) = 4.88$, $p<.03$; four (4, 8, 12), $x^2(1) = 7.50$, $p<.01$; and five (7, 14, 21), $x^2(1) = 6.97$, $p<.01$. For the number problems without the cheerios, significant group differences were found for problems two (6, 12, 18), $x^2(1) = 4.98$, $p<.02$; and three (3, 6, 9), $x^2(1) = 7.32$, $p<.01$. In all cases, the children with NLD achieved a greater success rate with the problems.

Finally, success on the five number problems with cheerios and the three without the cheerios was analyzed separately within each student group. In order to determine if one problem was significantly easier or more difficult than another, a cut value of .50 was used. When the binomial was applied to each of the five number problems (2-tailed, $p<.05$) with the cheerios, problems one (1, 2, 3), four (4, 8, 12), and five (7, 14, 21) were easier for the children with NLD whereas only problem one was easier for the children with LD. When the same test was run for

the three number problems without cheerios, problem three (3, 6, 9) was easier for the children with NLD only. In contrast, for the children with LD problem two (6, 12, 18) approached significance ($p<.06$) because it was more difficult than the other two problems.

WJTA-R

Qualitative Analysis

Calculation. For children in both groups who achieved at the first three levels on the fish task (i.e., additive structures of thought), 69% of the children were able to compute at least one problem with a one-digit multiplier while 53% additionally solved division problems with one-digit divisors. Twenty-one percent of the children computed problems with two-digit multipliers and divisors. (The number problem 120/12 did not count.) All children at the transitional level (IV) solved simple multiplication problems and all but one were additionally successful with one-digit division problems. Thirty-three percent of the children solved problems with a two-digit multiplier (two children had minor calculation errors) and two of these children were additionally successful with two-digit division. For children who achieved at levels V through VII which represented multiplicative thought structures, all solved simple multiplication problems, and all but one child also achieved success with one-digit divisors. For the problem with a two-digit multiplier, 87% of the children showed the ability to solve for these problems (seven of these children made minor calculation errors) while 65% also solved problems with a two-digit divisor.

Applied problems. Three problems that are meant to be indicative of use of multiplication and division on applied problems were looked at to determine how children were solving the problems. These problems are as follows: (a) Three people each have four dollars. How much

money do they have all together?; and (b) If Juanita saved a dime each day for one week, how much money would she have at the end of that week?

Not all children were given all problems due to basal level or ceiling performance levels. For children who achieved at the first three levels of the fish task, 52% of the children who were given the first two problems stated they didn't know or added. The remaining 48% provided a multiplication algorithm for at least one problem. Ninety percent of children at the transition level (level IV) provided an algorithm in multiplication for the first two problems, and 91% of the children at levels V through VII stated a multiplication fact when those problems were given.

Quantitative Analysis

Table 6 presents descriptive information on children's performance on the calculation and applied problems test of the WJTA-R by grade. On the calculation test, there was a misprint on the protocols in which an easier subtraction problem was replaced by a more difficult one. This error was detected only after all of the children were tested and the protocols scored. This misprint may have inflated the test results slightly for a small number of children on the calculation test. The mean standard score (standard deviation in parentheses) on the calculation test for children with NLD in grades 2, 4 through 7 respectively was approximately at grade level expectancy: 100(0.0), 101(14.9), 103(14.0), 104(8.8), 102(6.7), 102(10.0). In contrast, the mean standard score for children with LD tended to be slightly below grade level expectancy for the same grades respectively: 97(1.4), 104(0.0), 94(15.4), 90(7.0), 94(9.6). An ANOVA with grade level (5) by student group (2) as factors on the dependent variable of calculation showed no significant main effects or interactions. However, the main effect for group approached significance ($p<.07$) where the children with NLD achieved higher (type III sums of squares).

(Insert Table 6 about here)

The performance pattern for both groups on the applied problems test was higher than on the calculation test. For children with NLD, their performance was generally over grade level expectancy: 113(9.8), 120(5.6), 114(9.9), 116(7.9), 103(5.6), for grades 2, 4 through 7 respectively. For children with LD, their performance was generally at or slightly above grade level expectancy for the same grade levels respectively: 101(3.5), 99 (10.6), 96(8.4), 105(7.5), 108(14.1). An ANOVA with grade level (5) by student group (2) as factors on the dependent variable of applied problems showed a significant interaction $\underline{F}$ (4, 59) = 3.15, $\underline{p}$<.02. This interaction was disordinal because the scores of the children with NLD were much higher than the children with LD at the fifth grade level, but lower than the children with LD at the seventh grade level. However, the high performance of one LD child skewed the mean performance at the seventh grade level as noted by the large standard deviation (see Table 6).

DISCUSSION

There are three main findings of this study. The first of these findings is that children with LD evidenced the same developmental sequence in the evolution of ratio and proportion structures as children with NLD. Further, the inclusion of four BSI children in the LD group did not alter performance patterns between the LD and control children. However, a significantly greater number of children in the LD group used the less coordinated additive structures of logical-mathematical activity to solve the fish problems as compared to their peers with NLD. Thus, the majority of children with LD failed to show the coordinated activity of second-order

logical-mathematical structures necessary to act on problems using multiplicative reasoning. None of the children in either group demonstrated formal proportional reasoning.

Secondly, the sameness of explicitly taught operational procedures that children expressed in computing number problems on the WJTA-R failed to reflect the quality of their logical-mathematical activity observed on the fish task; this same trend was noted in applied problems. The performance of all children on applied problems, in which performance is heavily dependent on language as traditionally taught, generally exceeded performance on computations and was approximately at grade level expectancy for LD children.

Finally, a number of students in both groups used the repeated action of splitting when first constructing multiplicative thought structures. This observation supports the assertion by Confrey (1994) and Confrey and Smith (1995) that there is a splitting-analytical component to the evolution of multiplicative structures. However, this splitting-analytical component did not dominate over the distributive-algebraic component of multiplicative structures. Rather, these two conceptual components appear necessary and interdependent in the evolution of more coordinated multiplicative structures which is consistent with Lamon's (1996) observations.

In this cross-sectional design, the children in both groups were coordinating increasingly more complex composite unit structures as a function of grade. Specifically, children identified as using additive thought structures evidenced a progression in thought from: (I) adding random number amounts across the fish; to (II) consistently adding a numerical sequence of +1 or +2; proceeded to (III) differentiating a numerical amount to feed B (+2) and C (+3); and finally to (IV) transitioning to multiplicative thought structures where they added 2 to B and 3 to C in at least one problem and attempted a multiplicative solution in at least one other problem. These

levels represented the dominant pattern in students' thinking as there were students both transitioning into and out of these various levels. The majority of students with LD (62%) were identified as being at one of these levels while only 30% of students with NLD evidenced additive constructions.

Due to this performance difference between the two groups, problems three (3, 6, **9**), four (**4**, 8, 12), and five (**7**, 14, 21) with cheerios were significantly easier for the children with NLD. Most likely the difference between the two groups for problem one (**1**, 2, 3) was not significant because it could be solved using additive structures. Further, for problem two (2, **4**, 6), those children using the additive strategy of +2 could state the correct solution without coordinating (i.e., reflectively abstracting) second-order composite groupings. On the three number problems without the cheerios, problems two (6, **12**, 18) and three (**3**, 6, 9) were more difficult for children with LD. Note that on problem three with and without cheerios (3, 6, **9**; **3**, 6, 9), neither the medium requested for problem solution (i.e., cheerios or number problem) or which fish was first fed (i.e., A or C) resulted in a different performance pattern between the two groups. However, three additional NLD and two additional LD children correctly solved the problem when A was fed first in the condition where they wrote a number problem.

The lack of ability for LD and NLD children with additive structures to place the cheerios into multiplicative groups reflects the inability of their ordering activity to coordinate the more complex composite unit structures of the problem. Specifically, most of these children reflected on the task by redistributing n 1-units of cheerios across one level of abstraction. Of the small number of children who did attempt to group the cheerios into more complex composite units, both the actions of splitting and the joining of two composite units to form the total were noted.

When the splitting action was used, the even number groups were split into twos while the odd number groups were avoided. This action supports the contention that the halving algorithm is a powerful split (Confrey, 1994; Lamon, 1996) which children intuitively prefer before its relationship to equality is fully understood (Pothier & Sawada, 1983). Children's use of both the splitting structure and the joining of composite unit groups while grouping the cheerios speaks to the interrelated activity of partitioning and unitizing in the construction of operational structures such that neither process dominates (Lamon, 1996). Further, in the justifications for grouping solutions provided by a small minority of children, they attended to both the evenness of groups as well as the ease of counting the composite units.

Approximately one-third of all students combined who used multiplicative thought structures used the strategy of doubling to solve at least one problem. Thus, although the students' operational structures enabled them to coordinate second-order relationships consisting of a three-tiered unit composition, they directed their attention to the repeated split of halving or doubling an invariant unit of two. Further, even if a correct numerical solution was derived, in at least one other problem the great majority of the children failed to distribute invariant unit groupings across the fish n times. However, these same children generally made invariant groupings for each individual fish even if these groupings resulted in a remainder.

As in the first four levels of additive thought, if children justified their choice for groupings, they attended to the evenness of groups and/or to the ease of counting in constructing their groupings. The protocol previously reported of a student (Grade 6, NLD) at level VI exemplifies the interdependent relationship between the counting of composite unit groups (distributive-algebraic) and the splitting of groups (splitting-analytical). Specifically, she

tentatively hypothesized possible composite unit structures but needed the action of the splitting structure to think through and alter her solution. Again, such behavior supports Lamon's (1996) contention that these components are both necessary activities for structures to expand and coordinate second-order proportional relationships.

The strategy of deriving the correct solutions but with a lack of logical consistency in grouping the cheerios (level VI) is indicative of pre-proportional reasoning. Specifically, although the children attend to the sameness of groupings for each individual fish, they do not abstract this invariant structural component (i.e., ratio unit) as being equivalent across the three fish simultaneously. As such, the amount fed to each fish becomes a new element in the system such that there is no necessity of distributing equal groupings across all of the fish n times simultaneously. The protocol provided of a student (Grade 6, NLD) transitioning into the level that followed doubling (VI) reveals just how difficult it is for students to understand a quotient as a ratio unit. Specifically, when asked to write a number problem to represent the groupings of the cheerios, this student progressed from believing that it makes no difference how the cheerios are grouped to putting them into an additive relationship (8+4=12). Through a series of rethinking disequilibrations created by the experimenter's questions, he came to the correct groupings. Only at that point was he able to accurately represent the components of the algorithm as they relate to the groupings made. However, his structures are weak because he became confused about the problems that followed without the cheerios. Thus, he needs more time to think through problem relationships at his current level before his structures are ready to fully reorganize themselves and gain stability at a higher-order level. Progress thus comes through the experience of conflict which is the "business of the actor" (Steffe, 1990, p. 107).

The above observations support the contention that even though children may get the correct answer, if there is lack of evidence that they coordinate the structural similarity on the two sides of the equation, there can be no claim for proportional reasoning (Lamon, 1993, 1994; Lesh et al, 1988; Piaget & Inhelder, 1975). Even the children who had all solutions correct, including the correct groupings of the cheerios, were quite tentative about the necessity for keeping the groupings as they were. Further, none of the children actually used proportional language to describe the relationship; rather, they provided their rationale with a multiplicative equation. However, the fact that children at the highest level (VII) did evidence the use of relative thinking by relating a ratio unit across groups n times simultaneously when grouping the cheerios, does provide evidence for what Lamon (1993) referred to as qualitative proportional reasoning. This type of reasoning characterizes children who use ratio as a unit as well as the necessary element of a degree of relative thinking. In time, the reorganization and expansion of these thought structures through conflict leads to development of proportional reasoning.

If children's logical-mathematical structures are not coordinated enough to assimilate the second-order relationships presented in the counter-suggestion, they are unable to perceive it as a better solution or to provide a satisfactory explanation as to why the counter-example is better. In other words, there is no significant contradiction aroused to search for a plausible explanation. Recall the quality of response to the counter of two children at level IV (grade 5, LD & NLD). Specifically, the thinking activity of the child with LD, who was transitioning into this level, wandered from reflecting on the logical orders of the problem to an unrelated topic. In contrast, the thinking activity of the NLD child transitioning out of this level assimilated the multiplicative orders provided to her. As a result, her current orders of activity were disturbed enough to reflect

on the problems that followed using higher order coordinations as her structures were simultaneously reorganizing themselves onto more complex levels.

A significant observation related to this point is that problems one (1, 2, 3), four (4, 8, 12) and five (7, 14, 21) with cheerios were easier for children with NLD than problems two (2, 4, 6) and three (3, 6, 9) that preceded the counter suggestion. This finding could suggest that enough of these children were transitional in their structures to overcome their additive or doubling tendencies by the disequilibration created in the counter. Some of the children with NLD who were transitioning into multiplicative structures or using the doubling strategy provided logical explanations as to why the counter was better. As such, they appeared to assimilate the more complex structures presented in the counter. One could argue, however, that part of the difference may also be due to the fact that the last two problems may have been easier because fish A was fed first, thereby making the problem more familiar with regard to multiplicative algorithms. Problem one (1, 2, 3) was easier for these children as it could be readily solved using additive structures. For the children with LD, only problem one was easier. As such, neither the counter-example nor the fact that fish A was fed first in the problems that followed, altered the performance of the majority these children. To achieve the higher levels, it is necessary for their logical-mathematical structures to evolve to the degree necessary to assimilate the second-order relationships inherent in multiplicative structures.

On the problems without cheerios, children with NLD found problem three (3, 6, 9) to be easier than the other two problems. In contrast, only a minority of children with LD (approximately one third) represented the problems using multiplicative equations. As such, none

of these problems are significantly easier for these children. Problem two (6, 12, 18) proved to be just difficult enough to make it significantly harder for this group than the other two problems.

Children's performance on the applied problems test of the WJTA-R appears to contradict how children can reason mathematically. Specifically, approximately half of the children in both groups who achieved at the first three levels on the fish task with cheerios (additive thought structures) used a multiplication algorithm when given problem formats (including language) similar to what is practiced in school. With specific regard to children with LD, the mean standard score performance for these students at all five grade levels in applied problems (i.e., approximately 100) could lead one to conclude that these children have no significant problems in the language necessary to understand grade level mathematics concepts as represented on this test. The performance patterns in applied problems between the groups varied between grades where seventh grade students with LD achieved higher than their NLD peers. In contrast, the fifth grade students with NLD significantly out-performed their peers with LD. The mean score of the seventh grade students with LD (n=6) is skewed due to the particularly high performance of one student. This student achieved at level VII on the fish task.

Although performance in calculation is somewhat lower than in applied problems for both groups, the standard score performance for children with LD is within one standard deviation of the mean and not significantly different from their peers with NLD (this difference did, however approach significance). Further, when children thought out loud as they computed problems on the WJTA-R, they all voiced highly similar procedures. But these borrowed procedures that reflect the thinking of another leave undetected children's self-generated thinking activity. This contention is supported by the finding that the majority of children in both groups

(69%) who used additive schemes on the fish problem computed at least one problem with a one-digit multiplier while approximately half (53%) additionally solved a problem with a two-digit multiplier on the WJTA-R. In fact, seven of the children who used additive structures of thought had to be shown an example of a what a number problem is on the first problem without cheerios. Many additional students required guided questioning to help them to represent their thinking using standard algorithms. It appeared that these children did not "realize" that number facts serve the purpose of representing and solving a unique problem using their logical constructions as the start point. As one student (grade 5, LD) stated after struggling with the first problem but then completing the second problem with ease (albeit using additive structures), "I get it now. Before I didn't know how to do those things."

If we look at the student performance from two different theoretical perspectives of how logical-mathematical knowledge is constructed, then the differences between the fish task and the WJTA-R do not contradict each other. Specifically, the performance on the WJTA-R demonstrates that we can show children higher-level strategies that they can "learn" through repetition and practice of similar formulas and strategies as suggested by Carnine (1991, 1993, 1997) and Bley and Thornton (1995). Because many of these children generated multiplication algorithms to language structures frequently drilled in school, it is reasonable to infer that they can be "trained" to use higher-order strategies on the fish problem by transmitting steps on another's logic and practicing those steps. The larger issue is why such a path would be pursued.

Recall the child at level II (Grade 5, LD) who thought that either addition or multiplication are correct to solve problems. Even though he knew the multiplication algorithms on a rote level, he was unable to place the cheerios into groups and could not reflect on his

learning activity with any significant depth. Such behavior is an example of the somewhat fragmented and inconsistent thinking patterns that children construct when adult logic is imposed on structures that cannot assimilate such patterns of ordered activity. When children have not yet constructed structures necessary to coordinate the elements of another's thinking with their own logical orders, then the borrowed thinking from the more knowing person consists of bits of static information. As such, learning is merely rote and removed from the intellectual activity of the child (Cobb, Wood, & Yackel, 1991; Hatano, 1988; Piaget, 1973; Smedslund, 1961), thus depriving children of the opportunity to further expand their biologically based structures of organizing activity. As a result, knowledge will not be generalized to problems whose format differs significantly from the context in which it was learned. "In memorizing this simulated cut-and-dried copy of the logic of an adult, the child is generally made to stultify his own vital logical movement. The adoption by teachers of this misconception of logical method has probably done more than anything else to bring pedagogy into disrepute..." (Dewey, 1933, p. 81).

In conclusion, this study provides support to the belief that proportional reasoning evolves by the recoordinations of forms and structures of mental activity onto higher-order levels as puzzlement is confronted and resolved. These structures appear less expanded in children with LD as compared to their peers with NLD resulting is less coordinated inferences. Further, explicitly taught "problem-solving" steps generally exceed children's logical orders, thus creating static states of knowledge that are not generalized to authentic tasks that lack sameness in presentation. Such a practice increases the potential for distortions in reasoning. Finally, when children are constructing logical-mathematical structures, the splitting-analytical and distributive-algebraic processes are present simultaneously and support each other's evolution. The findings

of this study are consistent with previous research that found the evolution of simpler multiplicative structures in children with LD to be delayed as compared to their peers with NLD (Grobecker, 1997).

There are however, limitations to this study that need consideration. First, only a small number of children in the lower grades were included. However, for those children who did participate, the results were similar to those reported by Clark & Kamii (1996). Secondly, because language is subordinated to children's ordering activity, I argued that the degree to which children could coordinate composite unit structures was the determining factor in placing children at level achieved in the fish task. Children's reasoning is relative to what their logical structures can coordinate. However, the relationship between organizing activity and language is controversial thus requiring more research in all areas of mathematics that examines the biologically based organizing activity in children with and without LD and its relationship to language. Finally, it would be helpful to work with more children in grades five through eight to determine if the doubling strategy is again observed and to better examine how children at the higher grades come to understand the invariance of the ratio unit across fish. By including older children, it may be possible to evidence the use of formal proportional reasoning. Perhaps for these older children, it would be beneficial to have them compare their solutions across the problems to determine the stability of the abstractions they are making.

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Table 1

Mean ages, IQ scores, Standard Score WRAT Reading Scores, and Number of Males/Females for Each Student Group

Grade	Mean Age	IQ	WRAT	Male	Female
Children with NLD					
Second	7.8(0.4)	-	107(9.2)	2	1
Fourth	9.9(0.8)	108(5.7)	109(7.1)	1	1
Fifth	11.2(0.3)	104(5.5)	101(7.4)	7	3
Sixth	11.8(0.4)	110(6.2)	109(6.5)	7	3
Seventh	12.6(0.3)	108(3.8)	110(8.2)	2	3
Children with LD					
Second	7.8(0.7)	96(9.9)	93(9.2)	1	1
Fourth	9.5(0.5)	102(4.2)	88(2.8)	1	1
Fifth	11.1(0.4)	94(3.7)	83(11.6)	6	2
Sixth	11.8(0.5)	104(8.7)	92(13.9)	9	2
Seventh	12.5(0.3)	103(9.4)	99(5.5)	5	1

Note. Standard deviations in parentheses.

Table 2

Number (and Percent) of Children at Each Grade Level Receiving Remedial Intervention in Mathematics, Reading, and Language Arts

Grade	Mathematics			Reading	Language Arts
	Support	BSI	Replacement	Replacement	
Second			1(50)	2(100)	2(100)
Fourth	1(50)			2(100)	2(100)
Fifth	2(30)		5(63)	8(100)	8(100)
Sixth		4(36)	7(64)	7(64)[a]	7(64)
Seventh	3(50)		2(33)	4(67)	4(67)

[a]Note that all classified students were in replacement education for reading and language arts.

Table 3

Number (and Percent) of Students at Each Grade Level that Achieved at Each of the Seven Levels on the Fish Task by Grade Level for Each Student Group

Children with NLD							
			Fish Level				
Grade	1	II	III	IV	V	VI	VII
Second		1(33)	2(67)				
Fourth			2(100)				
Fifth				5(50)	4(40)		1(10)
Sixth					1(10)	7(70)	2(20)
Seventh					1(20)	1(20)	3(60)
Children with LD							
Second	1(50)	1(50)					
Fourth			2(100)				
Fifth	2(25)	2(25)	3(38)	1(12)			
Sixth		1(9)		2(18)	4(37)	3(27)	1(9)
Seventh		2(32)		1(17)	1(17)	1(17)	1(17)

Table 4

Number (and Percent) of Students Achieving Success on the Number Problems with Cheerios

Children with NLD

	Number Problem				
Grade	1, 2, 3	2, 4, 6	3, 6, 9	4, 8, 12	7, 14, 21
Second	3(100)	0 (0)	0(0)	0(0)	0(0)
Fourth	2(100)	2(100)	1(50)	0(0)	0(0)
Fifth	9(90)	6(60)	5(50)	8(80)	7(70)
Sixth	10(100)	9(90)	10(100)	10(100)	9(100)[a]
Seventh	5(100)	3(60)	4(80)	4(80)	5(100)

Children with LD

Second	1(50)	0(0)	0(0)	0(0)	0(0)
Fourth	2(100)	0(0)	0(0)	0(0)	0(0)
Fifth	7(88)	2(25)	0(0)	1(13)	1(13)
Sixth	11(100)	9(82)	7(64)	6(55)	6(65)
Seventh	6(100)	3(50)	4(67)	4(67)	4(67)

[a]One student was not given this problem.

Table 5

Number (and Percent) of Students Achieving Success without the Cheerios

Children with NLD

	Number Problem		
Grade	5, 10, **15**	6, **12**, 18	**3**, 6, 9
Second	0(0)	0(0)	0(0)
Fourth	0(0)	0(0)	0(0)
Fifth	6(60)	5(50)	8(89)[a]
Sixth	8(80)	9(90)	10(100)
Seventh	4(80)	4(80)	5(100)

Children with LD

Second	0(0)	0(0)	0(0)
Fourth	0(0)	0(0)	0(0)
Fifth	1(13)	0(0)	2(25)
Sixth	6(55)	7(64)	7(64)
Seventh	4(67)	2(33)	4(67)

[a]One student was not given this problem.

Table 6

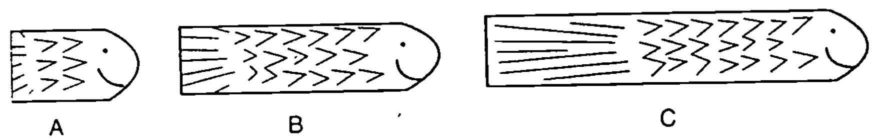
Mean Standard Scores on the Woodcock-Johnson (Revised)

Grade	Calculation	Applied Problems
Children with NLD		
Second	100.0(0.0)	112.7(9.8)
Fourth	100.5(14.9)	120.0(5.7)
Fifth	102.8(14.0)	113.5(9.9)
Sixth	103.9(8.8)	116.2(7.9)
Seventh	102.4(6.7)	102.8(5.6)
Children with LD		
Second	97.0(1.4)	100.5(3.5)
Fourth	104.0(0.0)	98.5(10.6)
Fifth	93.5(15.5)	95.6(8.4)
Sixth	90.0(7.0)	105.0(7.5)
Seventh	94.2(3.7)	107.8(14.1)

Note. Standard deviations in parentheses.

Figure 1.

The "fish" used in the task.



Notes

[1] Due to a concern by a reviewer that the inclusion of the four BSI children invalidated the results, all statistics were run without these children. None of the conclusions were affected, although the probabilities that the findings did not occur by chance, generally increased slightly. Because the performance patterns between the BSI and LD children were similar, the BSI children remained in the sample.

[2]Numerical indexes of cognitive ability and performance levels on the fish task are derived from two different cognitive constructs of what intelligent activity is. Specifically, IQ scores and cognitive scale indexes fail to reflect the degree of complexity in children's organizing activity which was the variable measured when assigning levels in the fish task.

[3] The author would like to acknowledge Frank Lawrence for his help with the statistical analysis.

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